Millersville University of Pennsylvania

Demographic Analysis of Nesting Bald Eagles in New Jersey using Mathematical Models

A Senior Thesis Submitted to the Department of Mathematics in Partial Fulfillment of the Requirements for the Departmental Honors Baccalaureate

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Abstract: Several mathematical and statistical models are built to analyze the growth and productivity rate of the New Jersey nesting bald eagle population. The effort works to quantify and project restoration of the bald eagle population from the deleterious effects of DDT. The best fitting model is found to be the logistic model and is used to calculate the population's asymptotic growth rate and carrying capacity. The logistic model is also used to predict the populations of the active pairs, successful pairs, and young produced bald eagle populations. The results indicate that the active bald eagle population in New Jersey increases with an intrinsic growth rate of 16.9% per year and has a carrying capacity of 280 active nesting pairs. According to this model, the active bald eagle population in New Jersey reached its maximum growth rate in 2014 and is projected to continue increasing at a slowing rate until it reaches its carrying capacity of active pairs by the year 2056.

Keywords: Bald eagles, Carrying capacity, Exponential model, Least squares regression, Logistic model, Stage-structured population model.

1 Introduction

The bald eagle (*Haliaeetus leucocephalus*) is a large raptor, easily distinguishable by its prominent white head feathers. This bird of prey has a long history with the Native Americans, and more recently can be associated with the United States of America as the national bird. The bald eagle has experienced considerable fluctuations in its population, from the overpopulation that lead to the establishment of a bounty in Alaska in 1917, to the dramatic decrease that resulted in the enactment of the Bald Eagle Protection Act in 1940, followed by being listed as Endangered in 1966 [4]. Due to extreme conservation measures, however, the bald eagle was removed from the Federal Endangered Species list in 2007 and is now considered one of the greatest recovery success stories [9].

Prior to the 1960s, the bald eagle had successfully inhabited the state of New Jersey. However, the use of strong chemicals such as DDT decreased the population to only 1 active pair by 1970. This pair nested in Bear Swamp (see Figure 1) which provided the tall trees adjacent to water typical of a bald eagle's chosen breeding site. The area's favorability is clearly shown by the density of nests observed in the region of Bear Swamp in 2018 [4]. In 1972, DDT, determined to cause thinning of avian eggshells, was banned by The United States Environmental Protection Agency (EPA) [14]. However, the lone New Jersey pair, contaminated with residual DDT, was nevertheless unable hatch young from 1976 - 1982 [6]. A 6-year hacking project then began with the hope of restoring the population in New Jersey. This project began in 1983 after a successful trial the year prior. This project consisted of removing the single egg laid by the active pair in 1983, and hatching it in captivity, before reintroducing it to the nest as a two-week old eaglet. This artificial hatching process continued with two new eaglets each year until completion of the project in 1988. The following year, a new adult female, uncontaminated by DDT, began nesting and was able to hatch eggs without human assistance, marking the end of the project. To augment the production from this original nest, 56 eagles, taken from nests in Manitoba, Canada, were also released in New Jersey beginning in 1982, up until 1990 [6].

In 1995, the project's goal of 10 new nests was surpassed, and in 2018, 185 active pairs were recorded in the state. This tremendous restoration, supported by 37 years of data, is the subject of this study. This data provides a comprehensive look at the New Jersey bald eagle population along with an understanding of the population dynamics of these large birds, and the long-term effects of hacking projects. The large amount of data obtained distinguishes this study from others due to historical "constraints of monitoring large areas over many years" [13].

Observational data from 1990 to 2018 are analyzed and fitted using several statistical and mathematical models. Section 2 introduces the data and shows several preliminary graphs. This section also shows the results of a linear regression analysis. Section 3 examines the exponential and logistic population models by comparing their R^2 values. Section 4 introduces the stage-structured population model, explains the life cycle of the female bald eagle, and shows the calculations of several vital rates. Finally, Section 5 summarizes and draws conclusions from the analysis. The goal of this study is to not only analyze the population data, but also to analyze how the hacking project has affected this population.

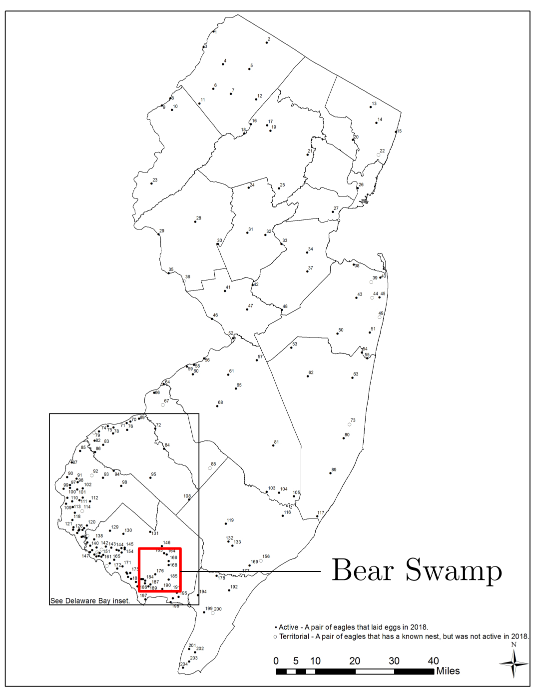


Figure 1: Nest distribution in New Jersey showing locations of known bald eagle nests in 2018 [41].

2 Statistical Analysis

2.1 Data Monitoring

Throughout this report, several terms are used. For purposes of clarity, note that: (1) "a pair of bald eagles" corresponds to "a single nest", and the terms are used interchangeably; (2) a pair of bald eagles is considered to be "active" if the nest contains at least one egg; (3) a pair is considered to be "successful" if it succeeds in fledging at least one eaglet; (4) "fledging" refers to both having hatched and survived through the age at which the young first leave the nest (usually between 8 and 14 weeks of age [4]); (5) "young produced" simply refers to the total young fledged in a given year; (6) "young / active" is the number of young produced per number of active nests and (7) "young / successful" is the number of young produced per number of successful nests. These terms are summarized in Table 1 for convenience.

Table 1: Terms

Active	A nest containing at least one egg.
Fledge	To successfully hatch and raise an eaglet to the age at which they leave the nest (8 – 14 weeks of age)
Success Rate	The number of successful pairs divided by the number of active pairs.
Successful	A nest that succeeds in hatching and raising at least one chick through the fledgling stage.
Young Produced	The number of young produced by all the successful nests in the year of study.
Young / Active	The number of young produced per number of active nests.
Young / Successful	The number of young produced per number of successful nests.

The data used in this report were obtained from the New Jersey Department of Environmental Protection: Division of Fish and Wildlife and are delineated by year in Table 2. The data were mainly gathered by volunteers, with observation of each known nest beginning in January of each year and lasting through fledging, or late July. Volunteers observed through binoculars at a distance of approximately 1,000 feet, recorded all data (e.g., number of birds and nesting behavior) and included any other observations relevant to nesting status. Volunteers also reported sightings of other bald eagles or other clues that might lead to a new nest location. All observational data were presented to the Endangered and Nongame Species Program (ENSP) and Conserve Wildlife Foundation (CWF) biologists and were then recorded following US Fish and Wildlife Service's monitoring plan. Every potential new discovery was investigated by ENSP staff and volunteers and an aerial survey may have taken place when evidence suggested a possible nest.

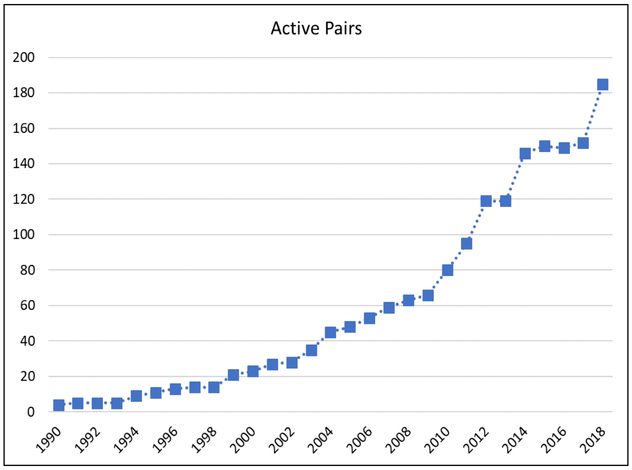
Table 2: Nest observational data

Year	Active	Successful	Success Rate	Young Produced	Young/ Active	Young/ Successful	Source
1982	1	1	1.00	1	1.00	1.00	[11]
1983	1	1	1.00	2	2.00	2.00	[24]
1984	1	1	1.00	1	1.00	1.00	[22]
1985	1	1	1.00	2	2.00	2.00	[23]
1986	1	1	1.00	2	2.00	2.00	[15]
1987	1	1	1.00	1	1.00	1.00	[16]
1988	1	1	1.00	2	2.00	2.00	[17]
1989	1	1	1.00	1	1.00	1.00	[18]
1990	4	3	0.75	5	1.25	1.67	[19]
1991	5	4	0.80	7	1.40	1.75	[26]
1992	5	3	0.60	4	0.80	1.33	[25]
1993	5	3	0.60	6	1.20	2.00	[20]
1994	9	6	0.67	12	1.33	2.00	[21]
1995	11	9	0.82	20	1.82	2.22	[50]
1996	13	9	0.69	14	1.08	1.56	[51]
1997	14	11	0.79	17	1.21	1.55	[52]
1998	14	10	0.71	17	1.21	1.70	[27]
1999	21	15	0.71	25	1.19	1.67	[53]
2000	23	18	0.78	30	1.30	1.67	[47]
2001	27	20	0.74	34	1.26	1.70	[42]
2002	28	22	0.79	36	1.29	1.64	[43]
2003	35	25	0.71	41	1.17	1.64	[44]
2004	45	32	0.71	54	1.20	1.69	[45]
2005	48	40	0.83	64	1.33	1.60	[46]
2006	53	47	0.89	82	1.55	1.74	[29]
2007	59	41	0.69	62	1.05	1.51	[30]
2008	63	50	0.79	85	1.35	1.70	[31]
2009	66	55	0.83	99	1.50	1.80	[32]
2010	80	42	0.53	69	0.86	1.64	[33]
2011	95	71	0.75	119	1.25	1.68	[34]
2012	119	100	0.84	165	1.39	1.65	[35]
2013	119	96	0.81	177	1.49	1.84	[36]
2014	146	115	0.79	201	1.38	1.75	[37]
2015	150	122	0.81	199	1.33	1.63	[38]
2016	149	129	0.87	216	1.45	1.67	[39]
2017	152	118	0.78	190	1.25	1.61	[40]
2018	185	121	0.65	172	0.93	1.42	[41]
Avg	60.10	46.10	0.75	76.62	1.27	1.69	
Std ±	54.41	42.45	0.08	69.80	0.20	0.17	

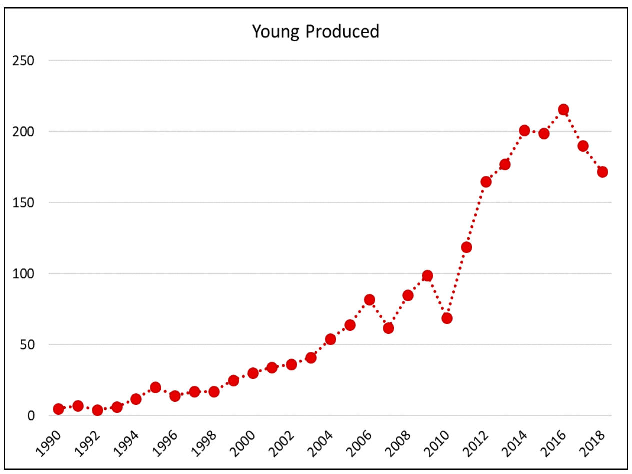
It is important to note the separation of the data between 1989 and 1990. This separation is due to the completion date of the hacking project. All calculations, including the averages and standard deviations at the bottom of Table 2, as well as all data fitting, only use data beginning in 1990 in order to best capture the natural growth after the completion of human interference.

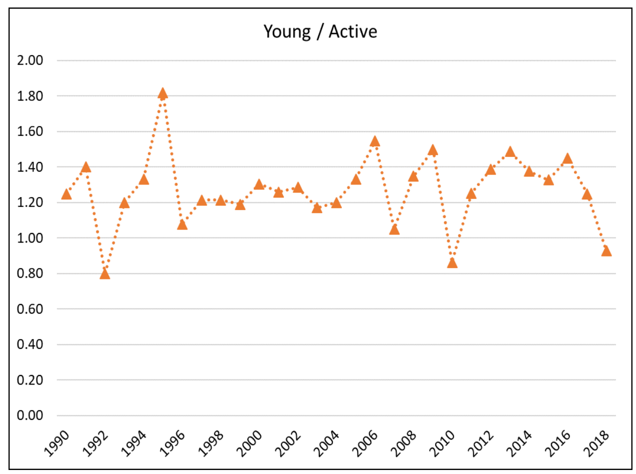
2.2 Observations

Notice all graphs in Figure 2 begin in 1990. As stated in the previous section, this best captures the natural growth of the species after the hacking project was completed.



(a) (b)

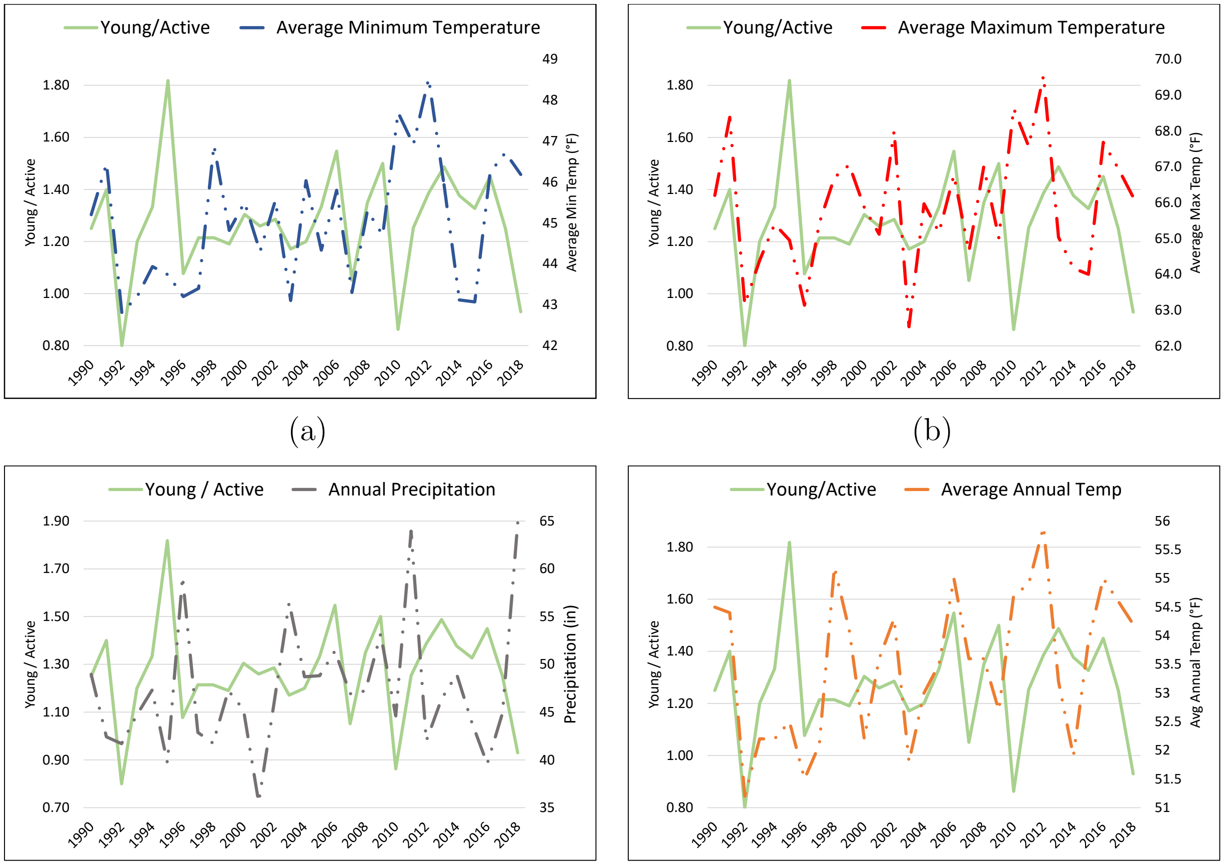




(c) (d)

Figure 2: Graphs of the observational data for active pairs (a), successful pairs (b), young produced (c) and young over active (d) from 1990 to 2018.

From Figure 2, it is evident that there has been a steady increase in the number of active pairs, as well as successful pairs, since 1990. The number of young produced also shows a general upward trend, with apparent decreases in 2007 and 2011. The graph of young produced per active pair shows no trend whatsoever, however, a study published in the Raptor Research and Management Techniques found that “year-to-year fluctuations in the nest success and productivity are common in raptors, and short-term decreases in productivity need not affect the long-term stability of populations” [49]. A statistical analysis of the young over active graph will be able to give an overall trend.



(a) (b)

(c) (d)

Figure 3: Weather trends: graphs of young / active vs. average minimum temperature (a), young / active vs. average maximum temperature (b), young / active vs. annual precipitation (c) and young / active vs average annual temperature (d) from 1990 to 2018 [28].

The graphs in Figure 3 explore the potential influence of weather on the number of young produced per active pair in a given year. The weather data are from Rutgers University and the average minimum and maximum temperatures refer to averages over the months of February through July, a typical breeding season for the bald eagle in this region [4]. Based on the fluctuation in years 1990 to 1998, 2010 to 2014, and 2016 to 2018, there seems to be a slight inverse relationship between temperature and productivity. Furthermore, the graphs of annual precipitation and annual temperature show an inverse relationship with the young over active data in the years 1994 to 1996, 2010 to 2012, and 2016 to 2018. The graphs shown in Figure 3 strongly suggest the influence of weather on the productivity of the bald eagle. Weather, however, is just one of the many factors that could affect the number of young produced by an active pair. Perhaps an area for further study would be on other factors that affect reproduction rates, such as habitat and available resources.

2.3 Linear Regression Analysis

To obtain an overall trend for each preliminary graph, linear regression with a 95% confidence interval was performed. Based on Figure 2, it can be determined, even before calculation, that a linear regression will not fit most of the graphs. This conclusion is confirmed by observing the following R^2 values in Table 3. R^2 is the coefficient of determination interpreted as the percent of variability of the population P that can be explained if the year t is known. A model is considered more accurate for R^2 values close to 1.

Table 3: Linear regression R^2 values

	R^2	Slope
Active Pairs	0.892	6.14
Successful Pairs	0.873	4.74
Young Produced	0.861	7.74
Young / Active	0.003	0.0013

Since these R^2 values are not close enough to 1, the linear regression fitting is not very accurate. However, the goal of completing the linear regression analysis was to see an overall trend for each graph, not to achieve the best fitting. Observing these trends is accomplished by examining Figure 4.

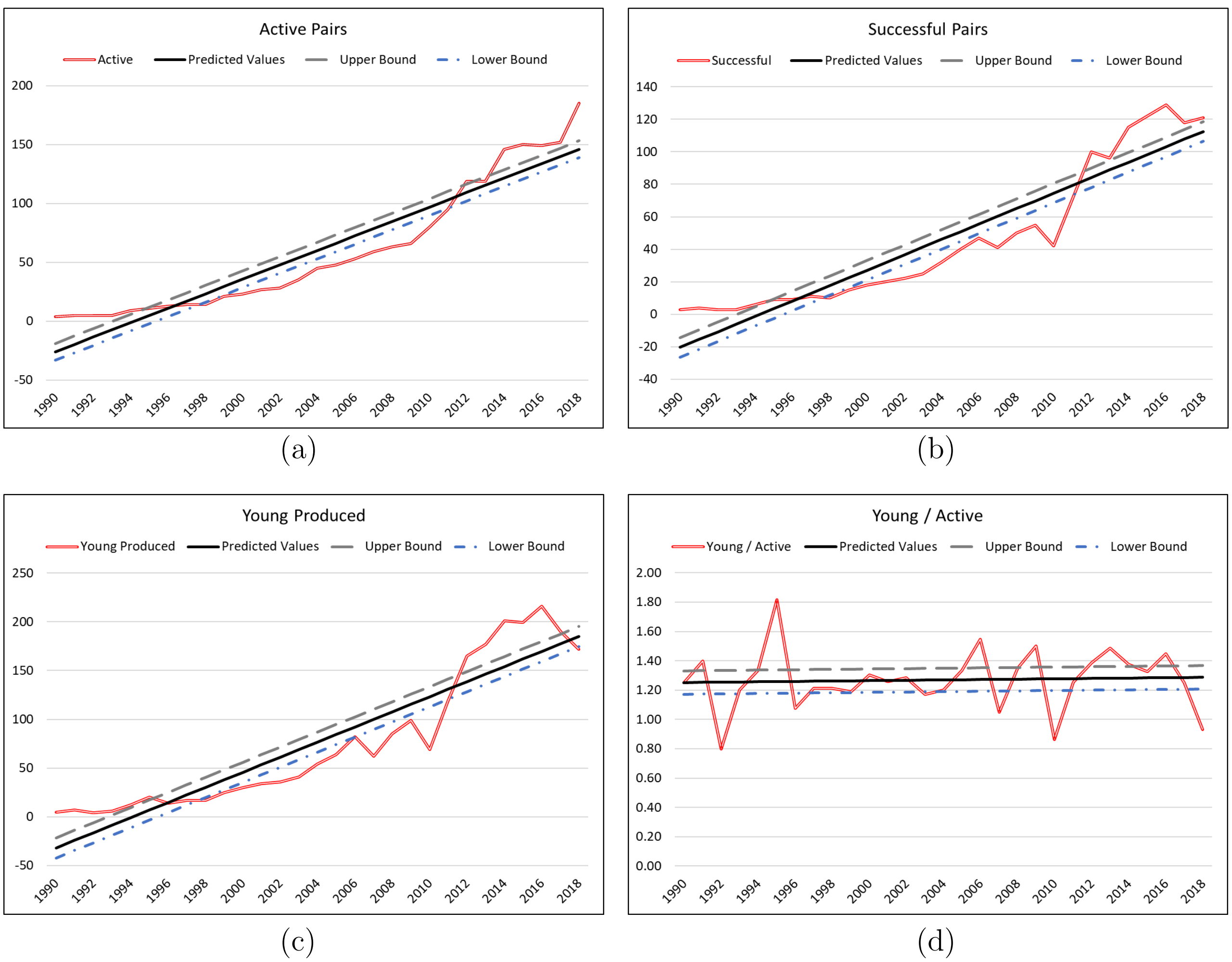


Figure 4: Linear regression trends: graphs of 95% confidence interval linear regression for active pairs (a), successful pairs (b), young produced (c) and young / active (d).

Although the linear regression does not give the most accurate fit for the data, it proves the original assumption made in Section 2.2 of an upward trend for both active and successful pairs as well as young produced. The slopes shown in Table 3 indicate the average increase in population for each year. Furthermore, while the young over active graph is still difficult to analyze visually, the linear regression calculated a slope of near zero. This result signifies that the ratio of young produced per active pair stays relatively consistent, even when a population is increasing.

3 Mathematical Models

In this section, the data will be fitted with two population models in order to establish a credible mathematical analog and a predictive capability for future populations.

3.1 Exponential Model

The first model examined here is the exponential population model. In this model, given a population (P), "the simplest hypothesis concerning the variation of population is that the rate of change of [P] is proportional to the current value of [P]" [2]. This leads to the differential equation,

$$\frac{dP}{dt} = rP, \tag{1.1}$$

where the constant of proportionality (r) is called the growth or decline rate, depending on whether it is positive or negative [2]. Here, r is considered positive, consistent with the upward trends visible in each preliminary graph in Figure 2. The population is increasing and r is referred to as the growth rate. Introducing the initial condition

$$P(0) = P_0, \tag{1.2}$$

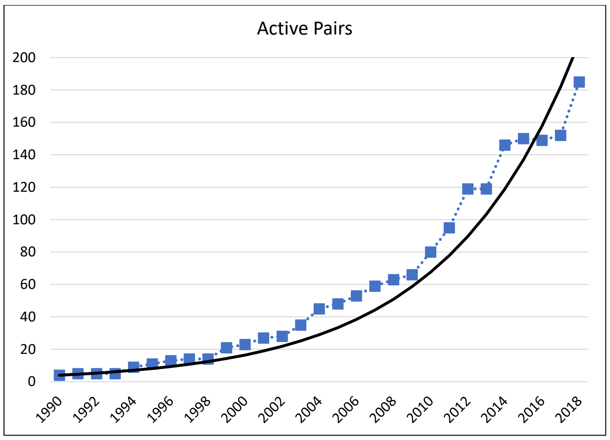
which refers to the original (initial) population at $t = 0$, yields the initial value problem

$$\begin{cases} \dfrac{dP}{dt} = rP \\ P(0) = P_0. \end{cases} \tag{1}$$

The system (1) can be solved to get

$$P(t) = P_0 e^{rt}. \tag{2}$$

Equation (2) is then used to calculate the population at each value of t, with t in years. In the subsequent calculations, $t = 0$ corresponds to the year 1990, with t incremented by one year at each iteration. Microsoft Excel's solver capability was used to calculate the growth rate (r) that minimized the sum of the squared errors, with no constraints imposed upon the exponential model. This model assumes that the given population will continue to grow indefinitely, as evident in Figure 5. It has been observed that the exponential model is reasonably accurate for short time periods [2], however, the ideal conditions necessary for uninhibited exponential growth, such as unlimited resources, cannot continue indefinitely. Eventually, limits on food or space will begin damping the growth rate [2]. The impact of such limitations will be further examined in the following section, when logistic growth is introduced.



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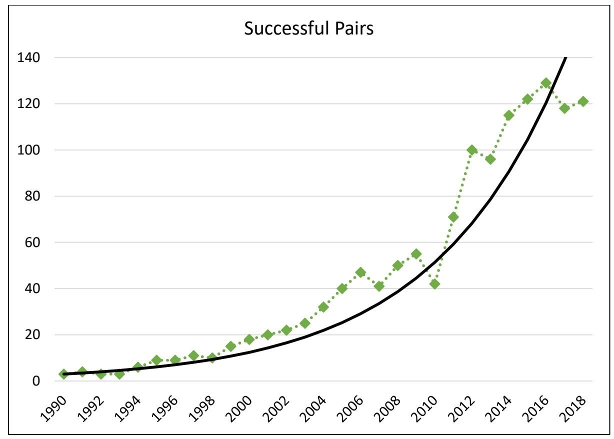

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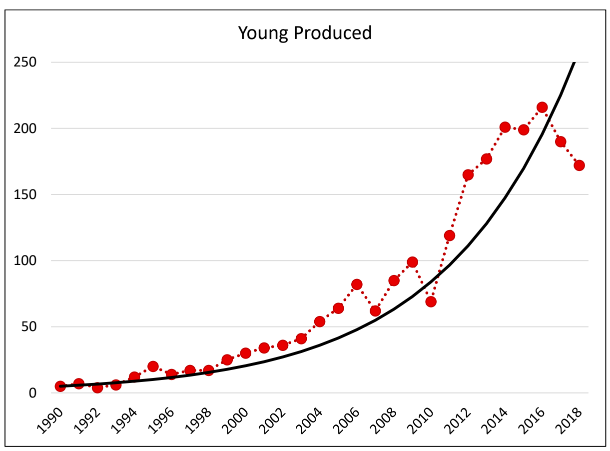


(c)

Figure 5: Exponential trends: graphs of exponential fitting for active pairs (a), successful pairs (b), and young produced (c) from 1990 to 2018.

Table 4: Exponential model fitting results

	R^2	Growth Rate (r)
Active Pairs	0.936	14.1%
Successful Pairs	0.893	14.2%
Young Produced	0.839	14.1%

Referring to the R^2 values in Table 4, the exponential model fits the data much better overall than the linear fitting. Looking at the graphs in Figure 5, however, the exponential model only fits the data well from 1990 to 2010, and fails to capture the more recent data. This observation is consistent with the previous statement regarding the accuracy of the exponential model. In order to better capture the last several years of data, a modified growth rate will be introduced in the next section.

3.2 Logistic Model

As mentioned in the previous section, there may be natural limitations imposed on resources that negatively impact the growth rate of any given population. The logistic population model attempts to better capture the realistic growth rate of a species by suggesting that the growth rate depends on the size of the population. In such circumstances, the r in Equation (1) is replaced with some function of the population ($h(P)$) to give

$$\frac{dP}{dt} = h(P)P, \tag{3.1}$$

where the function $h(P)$ is defined so that "$h(P) \cong r > 0$ when P is small, $h(P)$ decreases as P grows larger, and $h(P) < 0$ when P is sufficiently large" [2]. A simple function with these properties is $h(P) = r - aP$, where a is a positive constant. Substituting this function into Equation (3.1) gives the logistic equation defined as

$$\frac{dP}{dt} = (r - aP)\,P. \tag{3.2}$$

Rewriting Equation (3.2) and defining $K = \dfrac{r}{a}$ gives

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right). \tag{3}$$

In Equation (3), r is now referred to as the intrinsic growth rate, meaning the growth rate in the absence of any limitations [2]. This equation mimics the exponential model in the beginning of the growth phase, when the population size starts well below what is known as the environmental carrying capacity (K). The carrying capacity is the maximum number of a species that a habitat can sustain. While the exponential model assumes the population will continue increasing at the same growth rate for all time, the growth rate of the logistic model begins to slow at the inflection point. This point can be calculated by setting the second derivative equal to zero

$$\frac{d^2P}{dt^2} = 0,$$

then rewriting the second derivative to obtain

$$\frac{d}{dt}\left(\frac{dP}{dt}\right) = 0.$$

Replacing $\frac{dP}{dt}$ with Equation (3) gives

$$\frac{d}{dt}\left(r\left(1 - \frac{P}{K}\right)P\right) = 0.$$

Finally, differentiating with respect to t yields

$$r\left(1 - \frac{2P}{K}\right) = 0.$$

Solving for P gives

$$P = \frac{K}{2}.$$

After this point, $P = K/2$, the population continues to grow at a decreasing rate as it approaches its carrying capacity (K). An illustration of the solution is depicted in Figure 6.

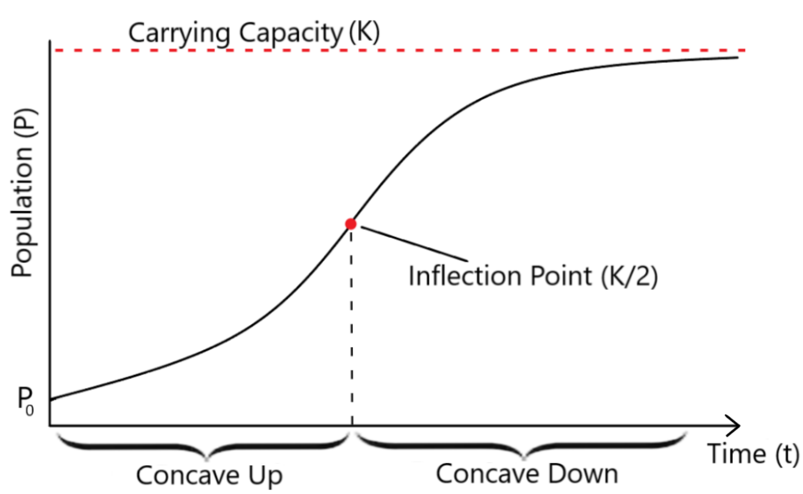


Figure 6: Logistic model example.

Figure 6 shows a general logistic model depicting a population that grows, at a changing rate, as it approaches its carrying capacity (K). The first half of the graph is very similar to the exponential model, however, after the population reaches exactly half of its carrying capacity (the inflection point), the growth rate begins to wane. On the graph, this point is observed by the change in concavity from positive to negative. The inflection point is labeled by a dot in Figure 6.

The first order differential equation (3) is paired with an initial condition to give

$$\begin{cases} \dfrac{dP}{dt} = rP\left(1 - \dfrac{P}{K}\right) \\ P(0) = P_0. \end{cases} \tag{4}$$

System (4) is then solved to give

$$P(t) = \frac{P_0 K}{P_0 + (K - P_0)e^{-rt}}. \tag{5}$$

As before, t is time in years, with $t = 0$ corresponding to the year 1990, $P(t)$ is the population at time t, and r is the intrinsic growth rate. Equation 5 is used to calculate future population levels at any year t. Values K and r are calculated using the solver capability of Excel to minimize the sum of the squared errors. The results are displayed in Figure 7.

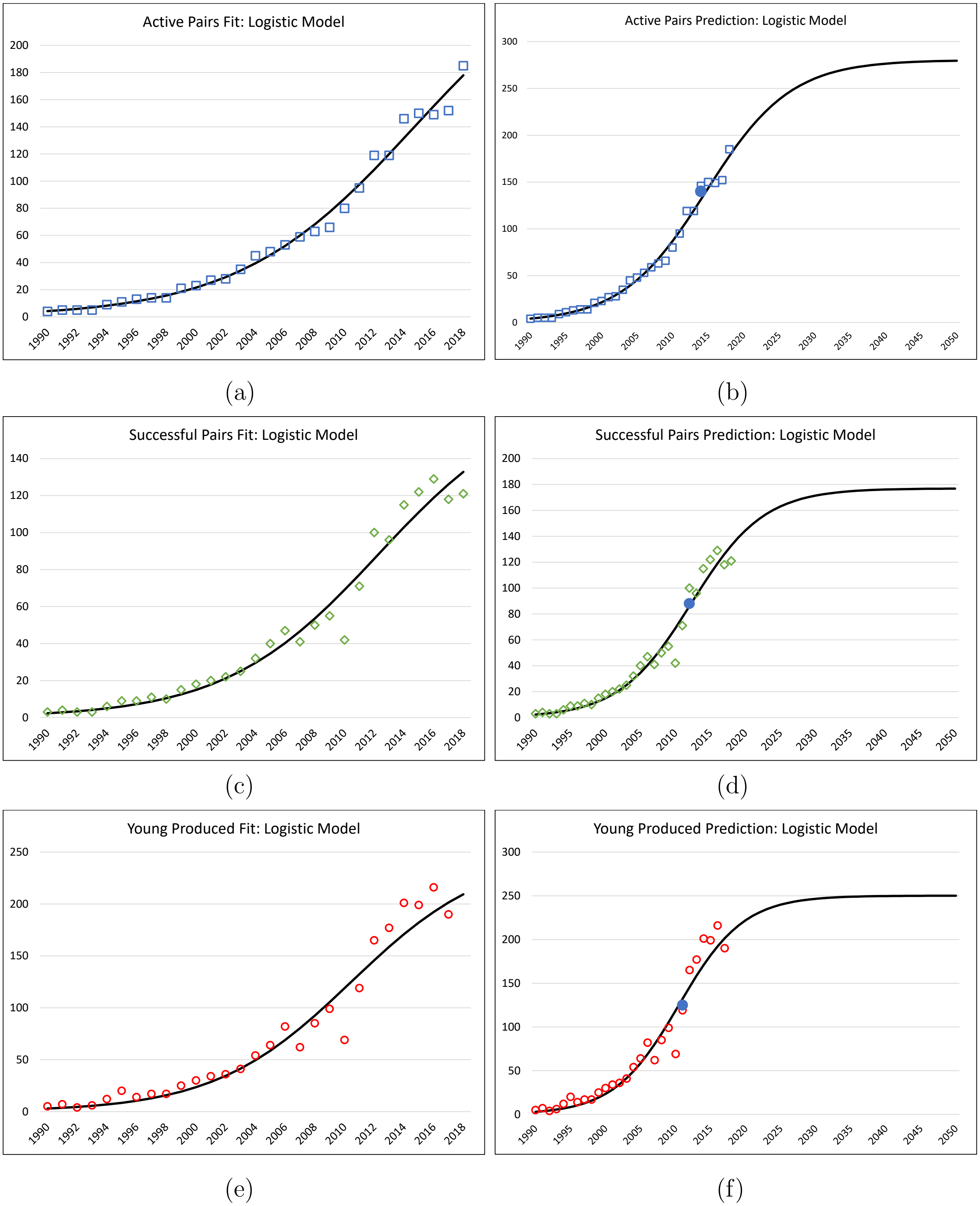


Figure 7: Logistic trends and predictions: graphs of active pairs logistic fitting from 1990 to 2018 (a), active pairs logistic prediction from 1990 to 2050 (b), successful pairs logistic fitting from 1990 to 2018 (c), successful pairs logistic prediction from 1990 to 2050 (d), young produced logistic fitting from 1990 to 2018 (e) and young produced logistic prediction from 1990 to 2050 (f).

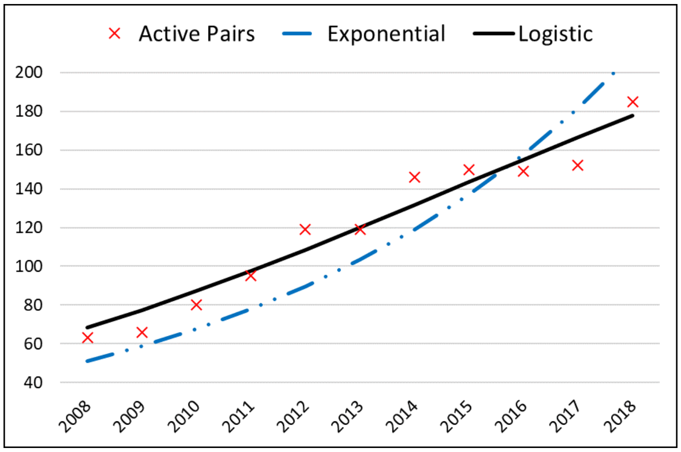
Figures 7 (a, c and e) display the fittings for active pairs, successful pairs and young produced, respectively, from 1990 to 2018. It is noted by comparing Figures 5 and 7 that the logistic model is more able to capture the last several years of data, where the exponential model failed. This observation is corroborated by the logistic R^2 values listed in Table 5. Figures 7 (b, d, and f) display predictions of each population until the year 2050 – a year chosen to clearly show how each graph eventually approaches its respective carrying capacity. More specifically, model predicts that New Jersey will reach its carrying capacity for active pairs ($K_a = 280$) in the year 2056. Dividing K_a by two gives $K_a/2 = 140$, which is the point of inflection, marked on Figure 7(b) with a dot. Each populations' respective inflection point is similarly marked with a dot. Referring back to Table 2, it is known that the population of active pairs reached 140 between the years 2013 and 2014. This means the population's growth rate has been declining since 2014. Again, this can be visually confirmed by referring to the graphs in Figure 7 and noticing the change in concavity that occurs at the inflection point $K/2$. It is important to note that each population's inflection point occurred almost simultaneously. This can be determined by reviewing the inflection points in Figures 7(b, d, and f) or by looking at Table 5. Based on the logistic calculations, the data is summarized below.

Table 5: Logistic Model Results

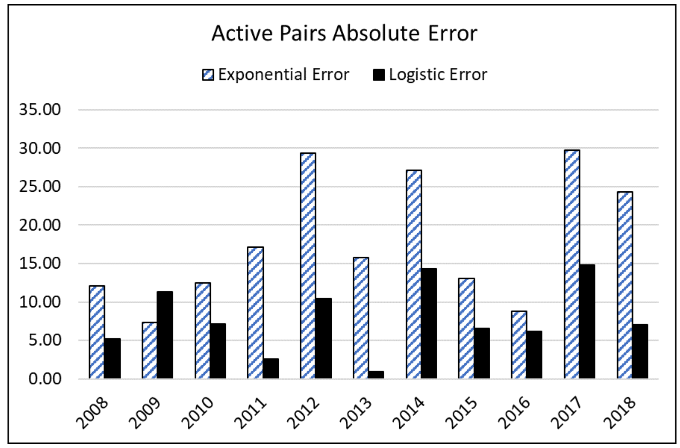
	R^2	Intrinsic Growth Rate (r)	Carrying Capacity (K)	Year inflection point was reached	Year carrying capacity will be reached
Active Pairs	0.989	16.9%	280	2014	2056
Successful Pairs	0.966	19.4%	177	2012	2062
Young Produced	0.946	21.7%	250	2011	2045

The R^2 values produced by the logistic model show a significant improvement in accuracy over the exponential model. This development is further examined in Section 3.3. As previously mentioned, it can be confirmed that the inflection point of each population was reached at a similar time, by referring to Table 5. In addition, each population is projected to reach its respective carrying capacity within a 20 year span of each other (young produced in 2045 and successful pairs in 2062).

3.3 Exponential and Logistic Model Comparisons



(a)



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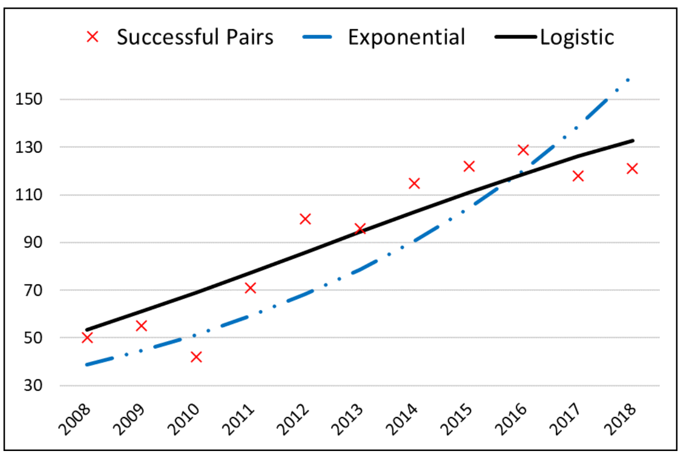

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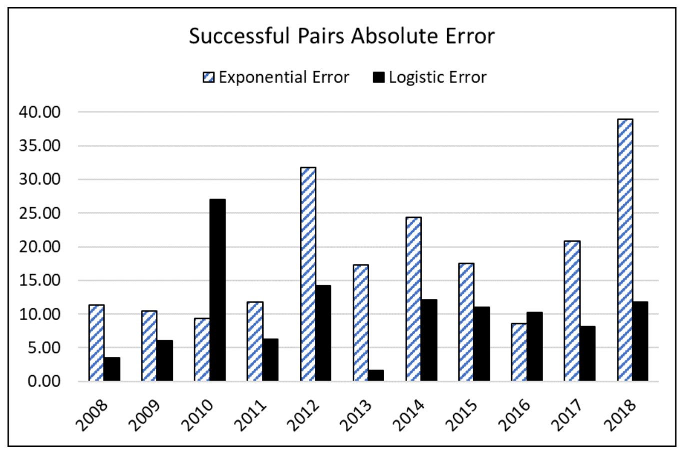

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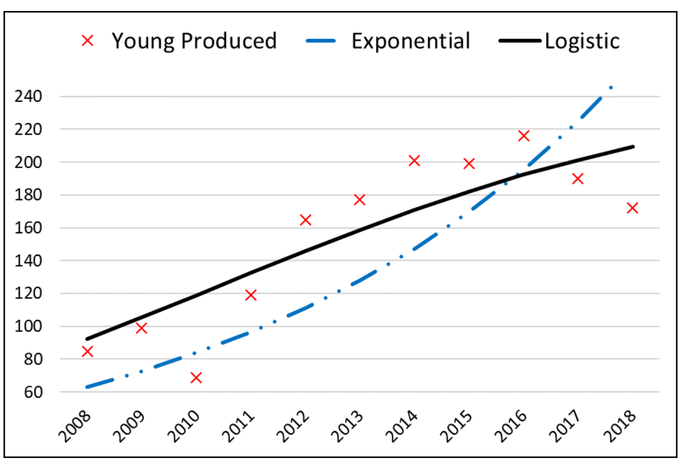

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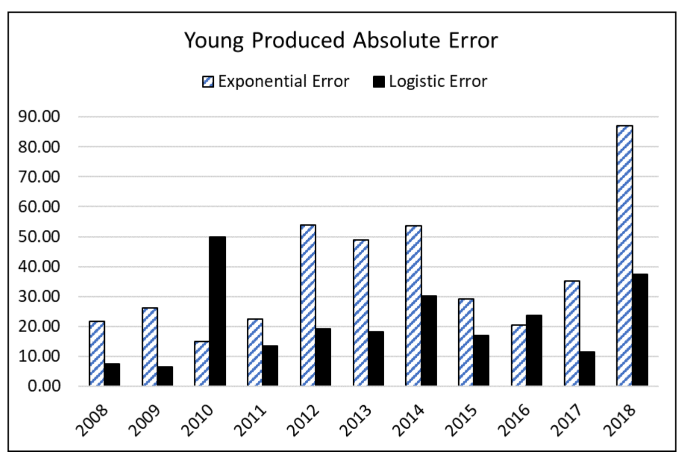


(f)

Figure 8: Figures of exponential model fitting vs. logistic model fitting for active pairs (a), absolute error graph of exponential model fitting vs. logistic model fitting for active pairs (b), exponential model fitting vs. logistic model fitting for successful pairs (c), absolute error graph of exponential model fitting vs. logistic model fitting for successful pairs (d), exponential model fitting vs. logistic model fitting for young produced (e), and absolute error graph of exponential model fitting vs. logistic model fitting for young produced (f). All figures showing the years 2008 to 2018

To better observe how the logistic model, rather than the exponential model, is able to capture the last several years of data, Figures 8 (a, c, and e) plot the observed data from Table 2 along with both the exponential and logistic model fittings for the years 2008 to 2018. Figures 8 (b, d, and f) show the difference in the absolute errors for both the exponential and logistic fittings at each data point. Overall, the results establish the logistic model as the superior fit of the data compared with that of the exponential model.

4 Stage-Structured Model

In this section, a stage-structured population model, the Lefkovitch matrix population model, is built to study the population dynamics of bald eagles. This model assumes the eagle population grows at a constant rate and reaches a stable stage distribution in which the proportion of individuals in each stage remain the same from one year to the next. The Lefkovitch matrix is used to predict this stable stage distribution and to calculate the population growth rate after this distribution is established. The stage-structured model takes into account the per capita survival, maturation, and reproduction rates of the female eagles. In Section 4.1, the female eagles are divided into three stages based on their life cycle. In Section 4.2, the least squares method is adopted to estimated the parameters in the matrix model. In Section 4.3, a demographic analysis is performed using the stage-structured model and sensitivities to vital rates are calculated.

4.1 Life Cycle

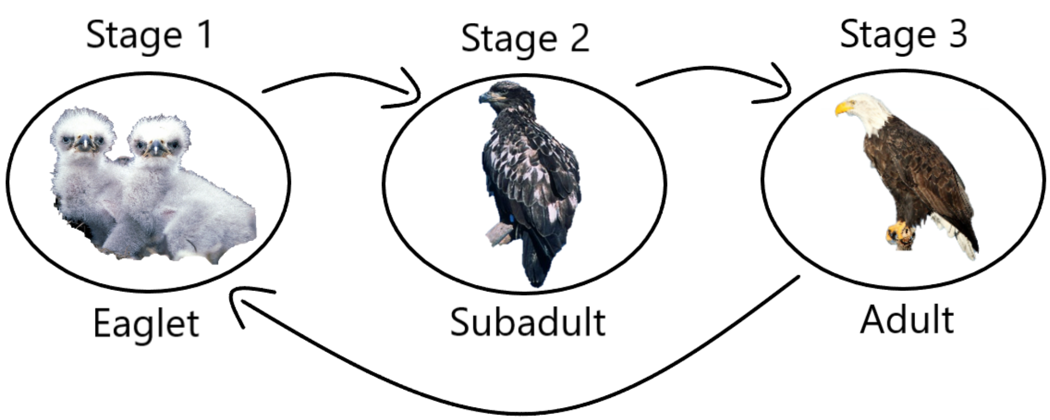


Figure 9: Life cycle of bald eagles (*Sources: Image of eaglet: https://www.tuckerhouse.com/blog/2015/04/bald-eagles-on-san-juan-island/ Image of subadult: https://birdsna.org/Species-Account/bna/species/baleag/introduction; Image of adult: https:pixabay.com/photos/bald-eagle-eagle-bald-perched-1247115/*).

The life cycle of the bald eagle (Figure 9) is divided into three stages for analysis, following the Cornell Lab Report [4]. "We estimated ages of subadult eagles (1-4 yr old) by plumage characteristics following McCollough. Adult eagles were those with definitive plumage and were assumed to be > 5 years old" [4],[12]. In this study, birds that fall into the first stage (< 1 year of age) are referred to as eaglets. The eagles in the first stage have successfully left the nest, but are still dependent on adult eagles for food [4]. The second stage birds (ages $1-4$ years of age) are called subadults. These birds are classified by their increased independence and their lack of the defining white plumage of an adult eagle. For the purposes of this study, it is assumed that subadults refrain from breeding. Finally, the third stage (≥ 5 years of age) is characterized by reproductive activity and the well-known white definition on their heads. It is assumed that the majority of adult eagles remain sexually active, and therefore remain in stage three, until the end of their life. The record age, 28 years, was recorded in Alaska, however, the lifespan of a bald eagle varies greatly depending on location [55]. The stage-structured model is implemented using the following system of equations:

$$\begin{aligned} N_1(t+1) &= bN_3(t), \\ N_2(t+1) &= S_1N_1(t) + (1-\gamma)S_2N_2(t), \\ N_3(t+1) &= \gamma S_2N_2(t) + S_3N_3(t). \end{aligned} \tag{4.1}$$

In this model, $N_i(t)$ refers to the number of active pairs in a given stage (i) of year (t), for $i = 1, 2, 3$. The individual survival rate of the first year eagles, the subadult eagles, and the adult eagles is represented by S_1, S_2, and S_3, respectively. The birth rate of female eagles is given by b, and γ is the proportion of subadult eagles that mature into the adult stage in one year.

Denote $\vec{\mathbf{N}} = [N_1, N_2, N_3]^T$. Rewriting Equations (4.1) in matrix form gives:

$$\vec{\mathbf{N}}(t+1) = A\vec{\mathbf{N}}(t), \tag{4.2}$$

where A represents the projection matrix. Then, the following system is obtained

$$\begin{bmatrix} N_1(t+1) \\ N_2(t+1) \\ N_3(t+1) \end{bmatrix} = \begin{bmatrix} 0 & 0 & b \\ S_1 & (1-\gamma)S_2 & 0 \\ 0 & \gamma S_2 & S_3 \end{bmatrix} \begin{bmatrix} N_1(t) \\ N_2(t) \\ N_3(t) \end{bmatrix}.$$

4.2 Parameter Estimation and Data Fitting

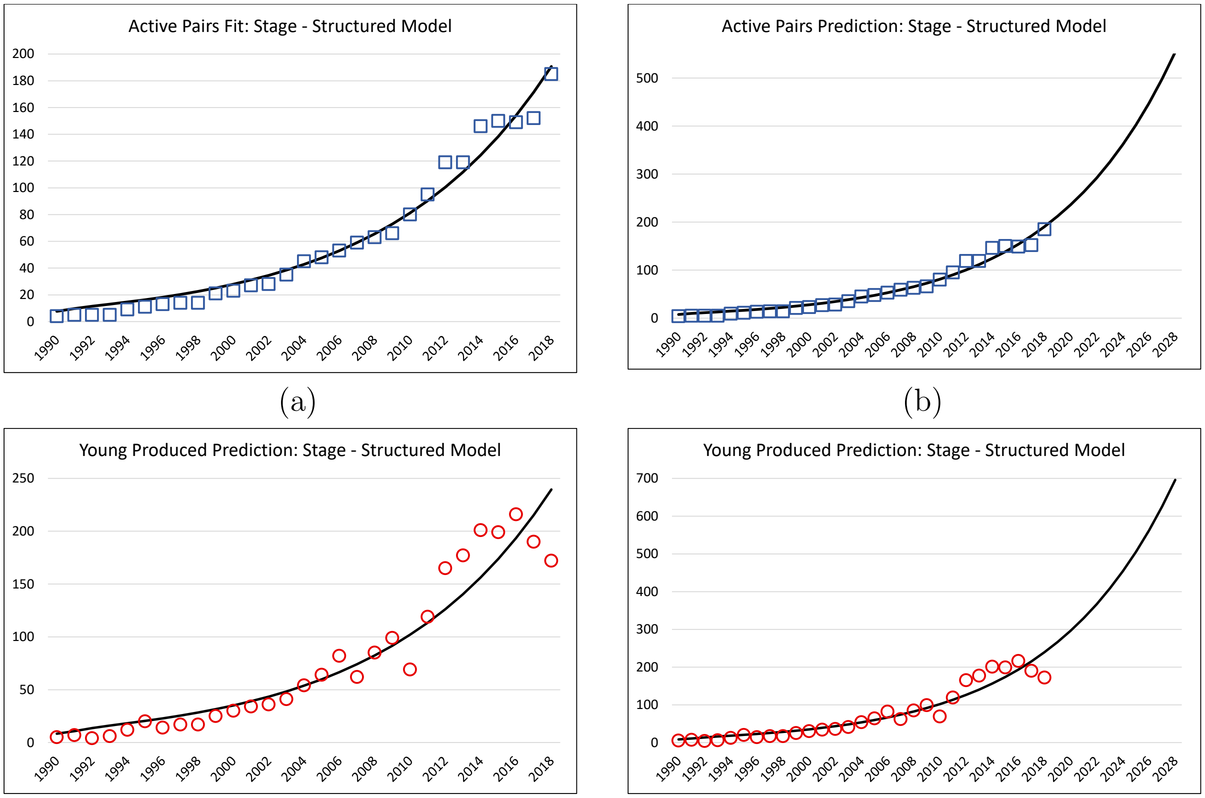
To implement the stage-structured model, several rates are required. The necessary rates are the birth rate (b), the survival rates of each stage (S_1, S_2, S_3), and the carryover rate between subadult and adult eagles (γ). From Table 2, average success rate is determined to be 75%, and average productivity is 1.27 young produced per active pair with a range of [1.07, 1.47]. This is close to pre-DDT nest success and productivity from the Chesapeake Bay at 79%, with 1.6 young produced per occupied territory [4],[3]. Based on the 1985 NJ Eagle Report, the sex ratio of the released birds was nearly 1 to 1 [23]. It is assumed that this sex ratio stays constant through the population increase and this assumption will be used to calculate the total population after the Lefkovitch matrix model calculates the female population.

The range for the survival rate of the first year bald eagle is estimated to be [0.05, 1.00]. In Alaska, the documented minimum first-year survival rate is 50% [10] and in the Chesapeake, survival of fledglings is 100% [5]. The rates for subadult and adults also come from the Chesapeake, [0.57, 1.00] and [0.63, 1.00], respectively [5]. The assumption was made that the survival rates were the same across both sexes [54], [1]. Based on the vital rates assumptions, the least squares method is used to fit the data of active pairs and young produced from 1990 to 2018 by the matrix model. The best fitting value for each parameter is listed in Table 6.

Table 6: Vital rates estimation

Vital Rates	Best fitting (least squares)	Interval Estimation
b	0.70	$[0.50, 1.00]$
S_1	0.50	$[0.50, 1.00]$
S_2	0.98	$[0.57, 1.00]$
S_3	0.92	$[0.63, 1.00]$
γ	0.22	$(0.00, 1.00]$

The approximated birth rate of female eagles is 0.70. Assuming the 1 to 1 sex ratio, this gives the total young produced by one active pair equal to 1.4 per year. This value is close to 1.27, the average clutch size, that is, the number of young per active pair, for New Jersey based on the data from Table 2. Using these rates, the graphs for the populations of active pairs and young produced are given in Figure 10. The stage-structured model is unable to calculate successful pairs since it considers only breeding females in its calculations.



(a) (b) (c) (d)

Figure 10: Stage-structured trends and predictions: graphs of stage-structured model fitting for active pairs from 1990 to 2018 (a), stage-structured model prediction for active pairs from 1990 to 2028 (b), stage-structured model fitting for young produced from 1990 to 2018 (c) and stage-structured model prediction for young produced from 1990 to 2028 (d).

From Figure 10, it is observed that the stage-structured model shows an ever-increasing growth, behaving similarly to the exponential model. This is unsurprising since the stage-structured model does not include the density-dependent vital rates that the logistic model employs.

4.3 Population Demographic Analysis

Applying the estimated vital rate values from Table 6 to the projection matrix A (4.3), gives

$$A = \begin{bmatrix} 0 & 0 & 0.6986 \\ 0.5000 & 0.7603 & 0 \\ 0 & 0.2161 & 0.9200 \end{bmatrix}. \tag{4.3}$$

To investigate the dynamics of the population of bald eagles, an important value to study is the asymptotic growth rate (λ). Under the assumption that the vital rates are invariant of time and environment, if $\lambda > 1$, the population will grow, while for $\lambda < 1$, the population will decrease. The dominant eigenvalue of A gives the population's asymptotic growth rate

λ [8]. The asymptotic growth rate is calculated to be $\lambda = 1.1126$ which means that the bald eagle population grows on average by approximately 11.26% per year. Note that the values calculated here fit nicely with the predictions made by Buehler et al. [5], where the eagle population in Chesapeake Bay, a nearby region, was predicted to grow by $5.8 - 16.6\%$ per year. The corresponding right eigenvector for λ provides the stable stage distribution (i.e., the proportion of individuals of each stage within the population) [8]. The calculated population stable stage distribution vector is given by (24.92%, 35.38%, 39.70%). This means when the population reaches a stable state, about 25% of the population will be eaglets, 35% will be subadults, and 40% will be adult eagles.

To further study the asymptotic growth rate (λ), a sensitivity analysis is conducted on each of the model's parameters to identify the life stage that contributes most to the population growth. This analysis can be beneficial, especially when considering how to better assist an endangered species.

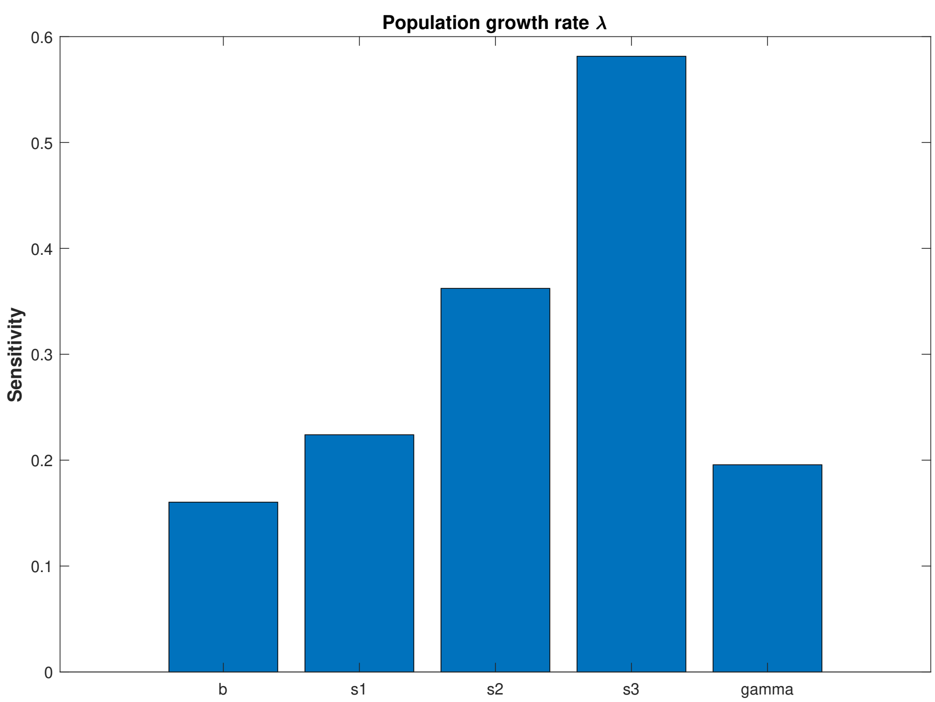


Figure 11: The sensitivity, to each vital rate, of the population growth rate λ

Figure 11 demonstrates the sensitivity of λ to each of the vital rates. The asymptotic growth rate (λ) shows the most significant sensitivity to the survival rate of the adult eagle, but is least sensitive to changes in the birth rate. More specifically, when S_3 is increased from 0.92 to 0.94, an increase of 2.17%, the value of λ will increase from 1.1126 to 1.1242, meaning the annual growth rate will increase from 11.26% to 12.42%.

The fundamental matrix (N), is one of the primary tools used in the analysis of population models. This matrix is defined by $N = (I - T)^{-1}$, where I represents the 3x3 identity matrix, and can be found by decomposing the projection matrix A into $A = T + F$ as follows

$$T = \begin{bmatrix} 0 & 0 & 0 \\ 0.5000 & 0.7603 & 0 \\ 0 & 0.2161 & 0.9200 \end{bmatrix} \tag{4.4}$$

and

$$F = \begin{bmatrix} 0 & 0 & 0.6986 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{4.5}$$

The matrix T describes the individual transition probability while the matrix F describes the individual fertility number, or the birth rate of the female eaglet. Then, the matrix N is defined as

$$N = \begin{bmatrix} 1.0000 & 0 & 0 \\ 2.0861 & 4.1723 & 0 \\ 5.6346 & 11.2692 & 12.5000 \end{bmatrix}, \tag{4.6}$$

where $N(i, j)$ gives the expected number of years that an individual starting at stage j will spend in stage i. The first column represents the female eaglet. On average, a female eaglet spends 1 year as an eaglet, roughly 2.1 years as a subadult, and 5.6 years as an adult. Life expectancy is one of the most important demographic characteristics of a population [7]. The expectancy vector for the bald eagle, $\vec{E}$, can be calculated by summing the columns of the fundamental matrix N in (4.6) [8]. Thus,

$$\vec{E} = (8.7207, \quad 15.4415, \quad 12.5000)\,. \tag{4.7}$$

The entries of $\vec{E}$ represent the life expectancy of each of the three stages of the bald eagle. For instance, the first entry of $\vec{E}$ implies that the life expectancy for an eaglet is about 8.7 years. The third entry determines that a mature adult will live, on average, an additional 12.5 years. The differences in the life expectancy between the eaglets and adult eagles is significant due to the change in mortality rates when eagles transition from one stage to the next.

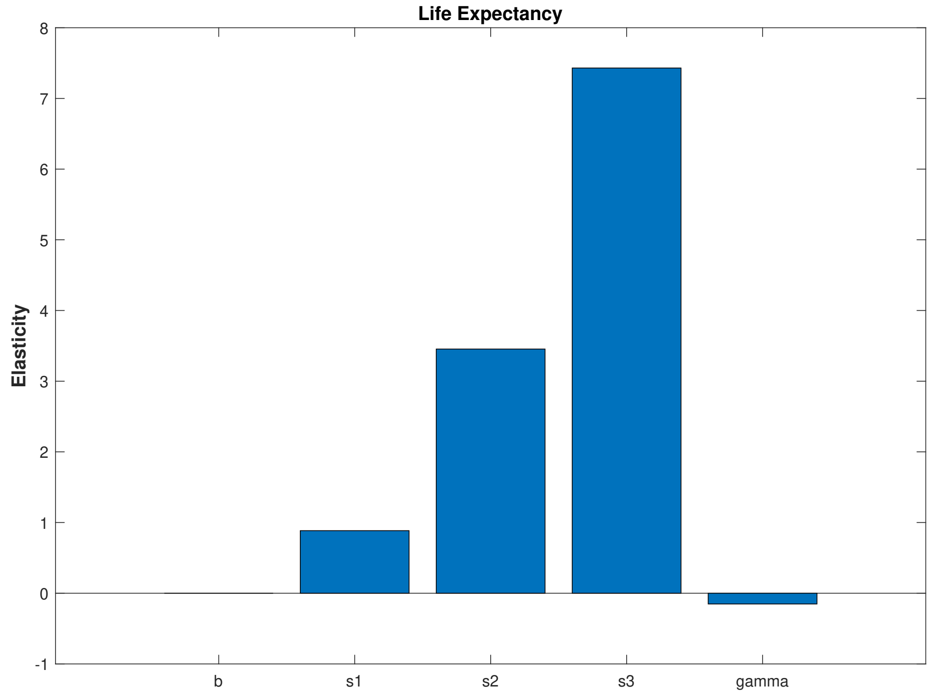


Figure 12: The graph of the elasticity of life expectancy for a female bald eagle to each of the vital rates.

The elasticity of life expectancy of a female eaglet with respect to the vital rates [8] is calculated and the results graphed in Figure 12. It shows that the life expectancy is most sensitive to changes in the survival rate of the adult (S_3) and shows a negative relationship with the maturation rate γ. This means that the faster the subadult matures into the adult stage, the shorter the life expectancy becomes. Furthermore, for every 1% increase in γ, the life expectancy decreases by a factor of less than one percent, and 1% increase in the value of S_3 would result in about 7.5% increase in the life expectancy. Also, the life expectancy is not affected by the birth rate.

5 Conclusion

Three population models were analyzed along with the New Jersey bald eagle population in this study; the exponential model, the logistic model, and the stage-structured model. Each model is graphed, along with the observed data from Table 2, in Figure 13. The exponential and logistic models were examined in Section 3, with the conclusion that the logistic model was more accurate than the exponential model. The stage-structured model was introduced in Section 4. A demographic analysis of the bald eagle nesting population was performed using the stage-structured model. The vital rates in the model were estimated using the data of active pairs and young produced from 1990 to 2018 by the least squares technique. These estimates were used to calculate the life expectancy for each stage and the asymptotic growth rate (λ) of the population. A sensitivity and elasticity analysis of the model parameters was then performed. For both life expectancy and the asymptotic growth rate, the survival rates have the greatest effect on these values, and they are much less affected by the maturation rate (γ) or the birth rate (b). These values are most sensitive to S_3, the survival rate of the mature adult. Specifically, any increase in the mortality rate of the mature females will have a more significant damaging effect on the population.

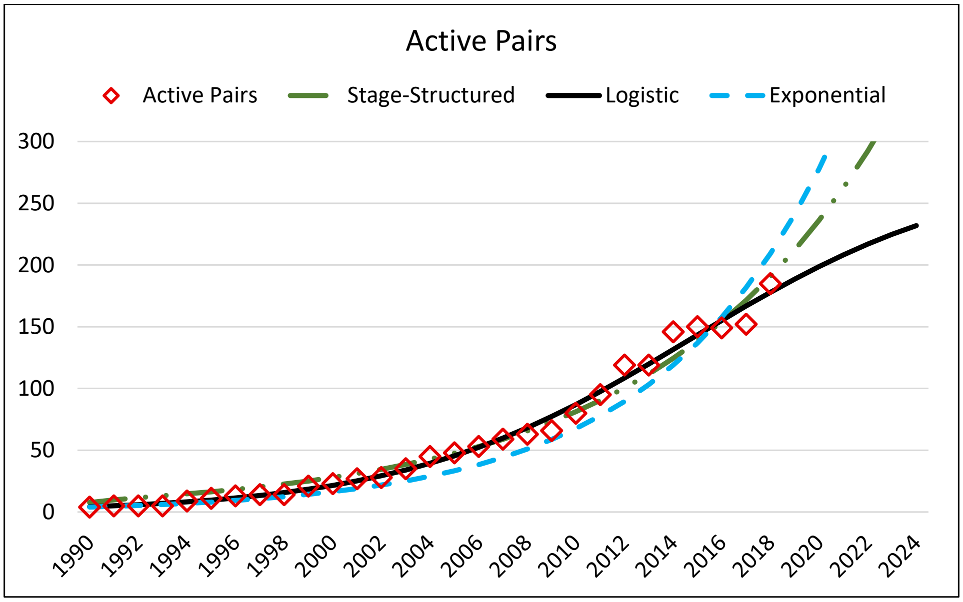


Figure 13: Graph of exponential, logistic, and stage-structured model fittings for active pairs population from 1990 to 2024.

From Figure 13, the stage-structured model and exponential model both predict that the active bald eagle population will continue growing at an increasing rate for the next few years while the logistic model predicts a decrease in the growth rate beginning after the year 2014.

Table 7: Exponential model, logistic model, and stage-structured model R^2 and squared error sum comparison

		R^2	Squared Error Sum (2008 to 2018)
Active Pairs	exp	0.904	4214.0
	log	0.989	878.2
	stage	0.977	1502.1
Successful Pairs	exp	0.862	4700.1
	log	0.966	1600.6
Young Produced	exp	0.839	20139.1
	log	0.946	6746.1
	stage	0.905	12342.2

Table 7 compares the R^2 values and squared errors for each of the three models. From Table 7, and the analysis presented in this study, the logistic model is the best fit for the data. Using this model, the populations of active pairs, successful pairs, and young produced can be predicted. These predictions of are summarized in Table 8.

Table 8: Predicted bald eagle populations (2019 to 2028) using the logistic model

	2019	2020	2021	2022	2023	2024	2025	2026	2027	2028
Active Pairs	189	199	208	217	225	232	238	244	249	253
Successful Pairs	139	144	149	153	157	160	163	165	167	169
Young Produced	216	222	227	231	235	238	240	242	243	245

The analysis presented in this study shows that the logistic model is the best fit for the population of bald eagles in the state of New Jersey. Using the logistic model, the population of active pairs, successful pairs, and the young produced was predicted and is summarized in Table 9, which can be used to better track the population in the near future. Also, this analysis concluded that the hacking project initiated 1983 was a success and would encourage others to implement such projects for other struggling eagle populations. Finally, the logistic model showed that New Jersey will reach its predicted carrying capacity of 280 active pairs by the year 2056.

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